Physics PI 2024-2025 solutions

Time 1

Task 1

1.1 $s_{x,y} = V_{0x,y}t + \frac{a_{x,y}t^2}{2}$; $|\vec{s}| = \sqrt{s_x^2 + s_y^2}$; $|\vec{\boldsymbol{s}}| = \sqrt{\mathbf{0^2 + 4^2}} = \mathbf{4\ cm}$

1.2 $V_x = \dot{x}$; $V_x = -2t + 4$; $V_x(t = 4) = -4$ m/s; $V_y = 1$ m/s; $\text{tg}\,\alpha = \frac{|V_y|}{|V_x|} = 0{,}25$; $\boldsymbol{\alpha} \approx \mathbf{14°}$

1.3 $V_x(t = 2{,}5) = -1$ m/s; $V_y = 1$ m/s; $\text{tg}\,\beta = \frac{|V_y|}{|V_x|} = 1$; $\beta = 45°$;

$\boldsymbol{a_\tau} = |\boldsymbol{a}|\cos\boldsymbol{\beta} = \mathbf{2\cos 45°} \approx \mathbf{1{,}41\ cm/s^2}$

Task 2

2.1 The bar is motionless, $F_x = 0 \Rightarrow \boldsymbol{F_f} = \mathbf{0}$

2.2 $ma_x = F_x - F_f$; $a_x = -\frac{t}{m} + \frac{3}{m} - \mu g$; $V_x = -\frac{t^2}{2m} + \left(\frac{3}{m} - \mu g\right)t$;

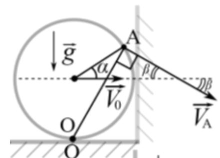
The bar stops ($V_x = 0$), at $t_1 = 2$ s. The block starts moving again when $-F_x = \mu mg$;

$t_2 = 3 + \mu mg$; $t_2 = 3 + 0{,}1 \cdot 2 \cdot 10 = 5$ s $\Rightarrow \boldsymbol{a(t = 4) = 0}$

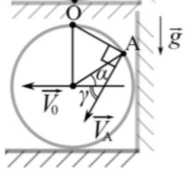
2.3 $a_x = -\frac{\tau}{m} + \frac{3}{m} - \mu g = 0$, at $\tau = 1$ c. $F_x(\tau) = 2$ N; $V_x(\tau) = -\frac{\tau^2}{2m} + \left(\frac{3}{m} - \mu g\right)\tau$;

$V_x(\tau) = 0{,}25$ m/s; $P_x(\tau)$=$F_x(\tau) \cdot V_x(\tau)$; $\boldsymbol{P_x(\tau)}$=$\mathbf{2 \cdot 0{,}25 = 0{,}5\ W.}$

Task 3



3.1 Point O is the intersection with the plane of the figure of the instantaneous axis of rotation of the hoop. The direction of the velocity of point A is perpendicular to the segment OA. From geometric considerations we find $\boldsymbol{\beta = 30°}$

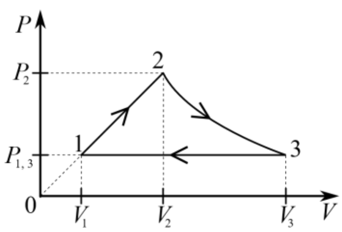


3.2 Point O is the intersection with the plane of the figure of the instantaneous axis of rotation of the hoop. The direction of the velocity of point A is perpendicular to the segment OA. From geometric considerations we find $\boldsymbol{\gamma = 60°}$

3.3 The hoop stops when the slipping stops:

$0 = V_0 + a\tau$, $a = -\mu g \Rightarrow \tau = \frac{V_0}{\mu g}$; $S = \frac{|a|\tau^2}{2} = \frac{V_0^2}{2\mu g} \Rightarrow \boldsymbol{\mu} = \frac{\boldsymbol{V_0^2}}{\boldsymbol{2Sg}}$; $\boldsymbol{\mu} = \frac{\mathbf{1^2}}{\mathbf{2 \cdot 0{,}5 \cdot 10}} = \mathbf{0{,}1}$

Task 4



4.1 $\frac{T_{\max}}{T_{\min}} = \frac{T_2}{T_1}$; $\frac{T_2}{T_1} = \frac{P_2V_2}{P_{1,3}V_1}$; $\frac{P_2}{P_{1,3}} = \frac{V_2}{V_1}$; $\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^2$; $\boldsymbol{\frac{T_2}{T_1}} = \mathbf{3^2}$=**9**

4.2 $P_2V_2 = P_{1,3}V_3$; $\frac{V_3}{V_1} = \left(\frac{V_2}{V_1}\right)^2$; $\boldsymbol{\frac{V_3}{V_1}} = \mathbf{3^2}$=**9**

4.3 $\eta = 1 - \frac{Q_c'}{Q_h}$; $Q_c' = \frac{5}{2}P_{1,3}V_1\left(\frac{V_3}{V_1} - 1\right)$; $Q_h = Q_{12} + Q_{23}$;

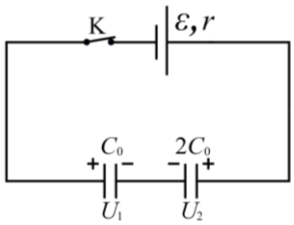
$Q_{12} = \nu \cdot 2R(T_2 - T_1)$, $Q_{12} = 2P_2V_2\left(1 - \frac{P_{1,3}V_1}{P_2V_2}\right)$, $Q_{12} = 2P_2V_2\left(1 - \left(\frac{V_1}{V_2}\right)^2\right)$;

$Q_{23} = A_T = \nu RT_2 \cdot \ln(n) = P_2V_2 \cdot \ln(n)$; $Q_\text{h} = P_2V_2\left(2\left(1 - \left(\frac{V_1}{V_2}\right)^2\right) + \ln(n)\right)$;

$\eta = 1 - \frac{\frac{5}{2}P_{1,3}V_1\left(\frac{V_3}{V_1} - 1\right)}{P_2V_2\left(2\left(1 - \left(\frac{V_1}{V_2}\right)^2\right) + \ln(n)\right)}$; $\eta = 1 - \frac{\frac{5}{2}\left(\frac{V_3}{V_1} - 1\right)}{\left(\frac{V_2}{V_1}\right)^2\left(2\left(1 - \left(\frac{V_1}{V_2}\right)^2\right) + \ln(n)\right)}$

$\boldsymbol{\eta = 1 - \frac{\frac{5}{2}(9-1)}{9\left(2\left(1-\frac{1}{9}\right) + \ln(3)\right)} \approx 23\%}$

Task 5



5.1 $q_1 = 2C_0\varepsilon$; $q_2 = 6C_0\varepsilon$; $\boldsymbol{\frac{q_2}{q_1} = 3}$

5.2 $\varepsilon = U_2 - U_1$ (1); from LCE: $q_2 - q_1 = q_1' + q_2'$,

$6C_0\varepsilon - 2C_0\varepsilon = C_0U_1 + 2C_0U_2$ (2). From (1) and (2): $U_2 = \frac{5}{3}\varepsilon$; $U_1 = \frac{2}{3}\varepsilon$;

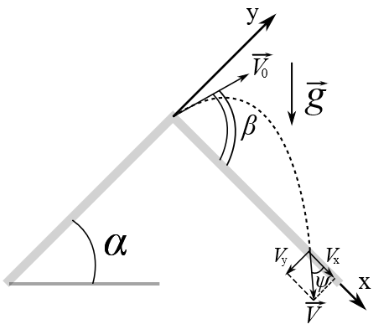
$|\Delta q| = |\Delta U_2| \cdot 2C_0$; $C_0 = \frac{|\Delta q|}{2|\Delta U_2|}$; $2C_0 = \frac{|\Delta q|}{\left(3\varepsilon - \frac{5}{3}\varepsilon\right)}$; $\boldsymbol{2C_0 = \frac{96}{\left(3\cdot 12 - \frac{5}{3}\cdot 12\right)} = 6}$ **μF.**

5.3 $Q = W_0 - W + A_{EMF}$; $W_0 = \frac{C_0(2\varepsilon)^2}{2} + \frac{2C_0(3\varepsilon)^2}{2} = 11C_0\varepsilon^2$; $W_0 = 11 \cdot 3 \cdot 12^2 = 4752$ mcJ; $W = \frac{C_0\left(-\frac{2}{3}\varepsilon\right)^2}{2} + \frac{2C_0\left(\frac{5}{3}\varepsilon\right)^2}{2} = 3C_0\varepsilon^2$; $W = 3 \cdot 3 \cdot 12^2 = 1296$ mcJ; $A_{EMF} = \Delta q\varepsilon$;

$A_{EMF} = -96 \cdot 12 = -1152$ mcJ; $\boldsymbol{Q = 4752 - 1296 - 1152 = 2304}$ **mcJ**

Time 2

Task 1



1.1 $T = \frac{2V_0 \sin\beta}{g\cos\alpha} \Rightarrow T_{max}$ when $\beta = 90°$, $\varphi = \beta - \alpha$, $\boldsymbol{\varphi = 60°}$

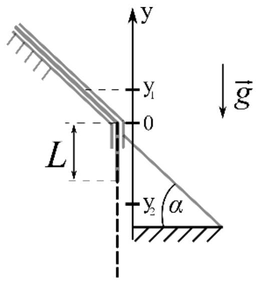
1.2 $V_x = V_0\cos\beta + gt\sin\alpha$; $V_y = V_0\sin\beta - gt\cos\alpha$;

$\operatorname{tg}\psi = \frac{|V_y(T_{max})|}{|V_x(T_{max})|}$, $\operatorname{tg}\psi = \frac{2-\sin\beta}{\cos\beta + 2\operatorname{tg}\alpha}$, $\operatorname{tg}\psi = \frac{\sqrt{3}}{2}$, $\boldsymbol{\psi \approx 40{,}9°}$

1.3 It can be shown that the maximum distance of flight along the slope is for a stone flew out at an angle $\beta_m = 45° + \frac{\alpha}{2} = 60°$;

$T_m = \frac{2V_0\sin\beta_m}{g\cos\alpha}$, $\boldsymbol{\frac{T_m}{T_{max}} = \sin\beta_m}$, $\boldsymbol{\frac{T_m}{T_{max}} \approx 0{,}87}$

Task 2



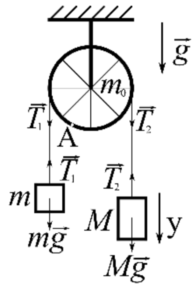
2.1 $F = Mg\left(\frac{L}{L_0} + \sin\alpha\left(1 - \frac{L}{L_0}\right)\right)$, where L_0 — length of the rope, $\boldsymbol{F = 30}$ **N.**

2.2 $a = g\left(\frac{L}{L_0} + \sin\alpha\left(1 - \frac{L}{L_0}\right)\right)$, $a = 6$ m/s²; $\boldsymbol{T = M\frac{l}{L_0}(a - g\sin\alpha)}$,

where l = 2 m — distance to the cross section from upper end of rope $\boldsymbol{T = 2}$ **N**

2.3 Initial coordinate of the center of mass of the rope: $y_1 = \frac{(L_0 - L)^2\sin\alpha - L^2}{2L_0} = 0{,}7$ m; final coordinate of the center of mass of the rope: $y_2 = -2{,}5$ m, $\boldsymbol{V = \sqrt{2g(y_1 - y_2)} = 8}$ **m/s.**

Task 3



3.1 $\boldsymbol{F = (M - m)g = 20}$ **N**

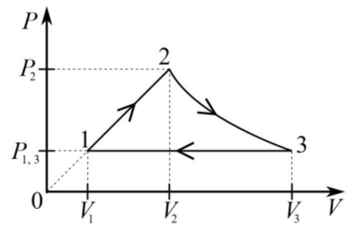
3.2 $-ma = mg - T_1$ (1), $Ma = Mg - T_2$ (2), $m_0 a = T_2 - T_1$ (3).

From (1),(2),(3) $a = g\frac{M - m}{m_0 + m + M} = 2$ m/s²; $\varepsilon = \frac{a}{R} = 2$ 1/s²; $\boldsymbol{S = \frac{\varepsilon t^2}{2}R = 4}$ **m.**

3.3 $N = T_1 + T_2 + m_0 g$, $T_1 = m(a + g) = 36$ N, $T_2 = m_0 a + T_1 = 40$ N

$\boldsymbol{N = 36 + 40 + 20 = 96}$ **N.**

Task 4



4.1 $\Delta U_{12} = \frac{3}{2}(P_2V_2 - P_{1,3}V_1)$, $A_{12} = \frac{1}{2}(P_2V_2 - P_{1,3}V_1)$, $\boldsymbol{\frac{\Delta U_{12}}{A_{12}} = 3}$

4.2 $\frac{V_3}{V_2} = \left(\frac{P_2}{P_{1,3}}\right)^{\frac{1}{\gamma}}$; $\frac{V_2}{V_1} = \frac{P_2}{P_{1,3}}$; $\frac{V_3}{V_1} = \frac{V_3}{V_2}\frac{V_2}{V_1} = \left(\frac{P_2}{P_{1,3}}\right)^{\frac{1+\gamma}{\gamma}}$; $\frac{1+\gamma}{\gamma} = 1{,}6$; $\frac{P_2}{P_{1,3}} = 4$

$\boldsymbol{\frac{V_3}{V_1} = 4^{1,6} \approx 9{,}2}$ **;**

4.3 $\eta = 1 - \frac{Q'_c}{Q_h}$; $Q'_c = Q'_{31}$; $Q_h = Q_{12}$; $Q_{12} = 2P_1V_1\left(\frac{P_2V_2}{P_{1,3}V_1} - 1\right)$; $Q'_{31} = \frac{5}{2}P_1V_1\left(\frac{V_3}{V_1} - 1\right)$

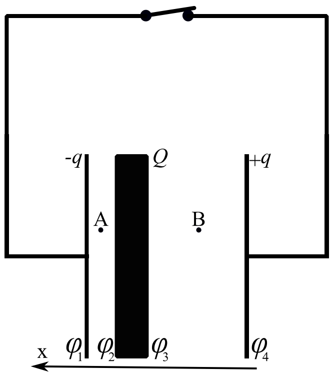
$$\boldsymbol{\eta = 1 - \frac{5\left(\frac{V_3}{V_1} - 1\right)}{4\left(\left(\frac{p_2}{p_{1,3}}\right)^2 - 1\right)} \approx 32\%}$$

Task 5

5.1 $C_0 = \frac{\varepsilon_0 S}{5d}$; $C_1 = \frac{\varepsilon_0 S}{d}$, $C_2 = \frac{\varepsilon_0 S}{3d}$; $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$; $C = \frac{\varepsilon_0 S}{4d}$; $\boldsymbol{\frac{C}{C_0} = \frac{5}{4} = 1{,}25}$

5.2 $E = \frac{Q}{2\varepsilon_0 S}$; $|U| = E \cdot 3d - E \cdot d = 2Ed$; $\boldsymbol{|U| = \frac{Qd}{\varepsilon_0 S} \approx 1130}$ **V.**

5.3 $E_{\mathrm{A}x} = \frac{1}{2\varepsilon_0 S}(2q + Q)$ (1), $E_{\mathrm{B}x} = \frac{1}{2\varepsilon_0 S}(2q - Q)$ (2),



where q — charge module on capacitor plates.

$\varphi_4 - \varphi_3 + \varphi_2 - \varphi_1 = 0$ (3); $\varphi_4 - \varphi_3 = E_{\mathrm{B}x} \cdot 3d$ (4),

$\varphi_2 - \varphi_1 = E_{\mathrm{A}x} \cdot d$ (5).

From (1), (2), (3), (4), (5) $\boldsymbol{q = \frac{Q}{4} = 2{,}5}$ **μC**

Time 3

Task 1

1.1 $V_1 = V_0 - a_\tau t_1$, $\boldsymbol{a} = \sqrt{\frac{V_1^4}{R^2} + (\mu g)^2} \approx \mathbf{2,2}$ **m/s²**

1.2 $V_2 = V_0 - a_\tau t_2$, $a_\tau = 1$ m/s², $a_n = \frac{V_2^2}{R} = 0{,}5$ m/s², tg $\alpha = \frac{a_\tau}{a_n}$; $\boldsymbol{\alpha \approx 63,4}$°

1.3 $\tau = \frac{V_0}{\mu g}$; $\alpha = \frac{V_0}{R}\tau - \frac{\mu g}{2R}\tau^2 = 25$ radians; $\boldsymbol{N = \frac{\alpha}{2\pi} \approx 3,98}$

Task 2

2.1 $|a_P| = \mu g \frac{m}{M}$, where m — puck mass, M — plank mass, μ — coefficient of friction of the puck on the plank, g — acceleration of gravity. $|\boldsymbol{a_p}| = \mathbf{1,5}$ **m/s²**.

2.2 $\boldsymbol{E_k = \frac{(m+M)V_0^2}{2} - \mu mgL = 3}$ **J**

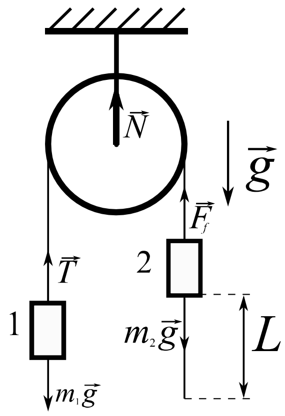
2.3 $L = 2V_0\tau - \frac{|a_{rel}|\tau^2}{2}$ (1); $|a_{rel}| = |a_{puck}| + |a_P| = 4{,}5$ m/s² (2),

where $|a_{puck}| = \mu g = 3$ m/s² — puck acceleration. We substitute the relative acceleration $|a_{rel}|$ and the initial data into (1), solve the quadratic equation and find τ — the time of the puck's movement along the plank from the moment the plank collides with the wall until the moment the puck flies off the plank, $\tau \approx 0{,}3$ s.

In time τ the right end of the plank travels the distance $\boldsymbol{l = V_0\tau - \frac{|a_p|\tau^2}{2} \approx 53}$ **cm.**

Task 3

3.1 $\boldsymbol{\frac{N}{F_f} = 2}$



3.2 $m_1 a_1 = \alpha m_2 g - m_1 g$. When $\frac{m_2}{m_1} = 5$, $a_1 = 0$,

where a_1 and a_2 — loads 1 and 2 accelerations.

$a_2 = g(\alpha - 1) = -8$ m/s². Time of movement of load 2 before slipping $\tau = \sqrt{\frac{2L}{|a_2|}} = 0{,}5$ s. The module of the velocity of load 2 at the moment of slipping from the thread $\boldsymbol{|V| = |a_2|\tau = 4}$ **m/s**

3.3 When $\frac{m_2}{m_1} = 6$, $a_1 = 2$ m/s² — load 1 moves up, $a_2 = -8$ m/s²;

$|a_{rel}| = |a_2| - |a_1| = 6$ m/s²; Time of movement of load 2 before slipping $\tau_1 = \sqrt{\frac{2L}{|a_{rel}|}} = \frac{1}{\sqrt{3}}$ s;

The velocity module of load 1 at the moment of slipping off the thread of load 2

$\boldsymbol{|V_1| = |a_1|\tau_1 \approx 1,2}$ **m/s.**

Task 4

4.1 $\boldsymbol{Q = \frac{5}{2}A = 5}$ **kJ,** Where A — work done by helium.

4.2 $\Delta \boldsymbol{m} = \frac{\mu P \Delta V}{RT_0} = \frac{\mu A}{RT_0} \approx \mathbf{11,6}$ **g,** where μ — molar mass of water, R — universal gas constant, $T_0 = 373$ K.

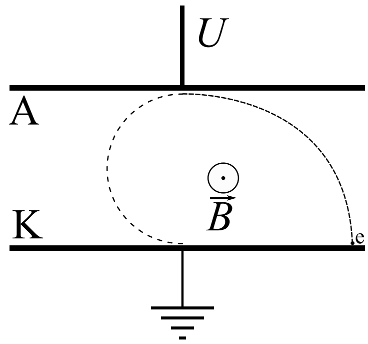
4.3 $\Delta \boldsymbol{U} = \boldsymbol{A} - \boldsymbol{\lambda} \Delta \boldsymbol{m} \approx \mathbf{-24}$ **kJ,** where λ — specific heat of evaporation of water.

Task 5

5.1 $\boldsymbol{E} = \frac{U}{d} = \mathbf{100}$ **V/m**, where d — distance between electrodes.

5.2 $\boldsymbol{V} = \sqrt{2\gamma U} \approx \mathbf{593}$ **km/s,** where γ — modulus of specific charge of electron.

5.3 $\boldsymbol{\tau} = \frac{\pi d}{2V} \approx \mathbf{26}$ **ns.** See the trajectory of the electron in the figure.



Time 4

Task 1

1.1 $x = V_0 t; y = \frac{at^2}{2} \Rightarrow \boldsymbol{V_0} = \sqrt{\frac{\boldsymbol{a}}{\boldsymbol{2k}}} = \mathbf{1}$ **m/s**

1.2 $V_x = V_0, V_y = at; \boldsymbol{V} = \sqrt{\boldsymbol{V_x^2} + \boldsymbol{V_y^2}} \approx \mathbf{2,2}$ **m/s**

1.3 $\mathrm{tg}\,\alpha = \frac{V_y}{V_x} = \frac{at}{V_0} = 2$; $\alpha \approx 63{,}4°$; $\boldsymbol{a_\tau} = \boldsymbol{a} \cdot \boldsymbol{\sin\alpha} \approx \mathbf{0,89}$ **m/s²**

Task 2

2.1 $\boldsymbol{a} = \frac{\boldsymbol{F_0}}{\boldsymbol{m}} = \mathbf{4}$ **m/s²**.

2.2 Projection of the puck impulse onto the axis OY: $p_y = \frac{F_0 t_0 \pi}{2}$; projection of the puck velocity onto the axis OY: $V_y = \frac{F_0 t_0 \pi}{2m} \approx 12{,}6$ m/s; $V_x = 10$ m/s; $|\vec{\boldsymbol{V}}| = \sqrt{\boldsymbol{V_x^2} + \boldsymbol{V_y^2}} \approx \mathbf{16,1}$ **m/s.**

2.3 $V_y(t_0) = \frac{F_0 t_0 \pi}{4m} \approx 6{,}28$ m/s; $V_x(t_0) = 10$ m/s; $V = |\vec{V}(t_0)| \approx 11{,}8$ m/s;

$\mathrm{tg}\,\alpha = \frac{V_x(t_0)}{V_y(t_0)}$, $\alpha \approx 57{,}8°$; $a_n = a \cdot \sin\alpha \approx 3{,}4$ m/s²; $\boldsymbol{R} = \frac{\boldsymbol{V^2}}{\boldsymbol{a_n}} \approx \mathbf{41,1}$ **m**

Task 3

3.1 $\boldsymbol{a} = \boldsymbol{g}\frac{\boldsymbol{m}}{\boldsymbol{M+m}} = \mathbf{5}$ **m/s²**, where M — plank mass, m — load mass.

3.2 $A_f = \frac{\mu g M l}{2} = mgl \ \Rightarrow \frac{\boldsymbol{M}}{\boldsymbol{m}} = \frac{\mathbf{2}}{\boldsymbol{\mu}} = \mathbf{10}$, where μ — coefficient of friction of the plank on the rough part of the horizontal surface.

3.3 V_{max} at $a = 0 \ \Rightarrow$ The plank rides onto the rough part of the horizontal surface by

$x = \frac{l}{\mu \frac{M}{m}} = 0{,}625$ m. From the theorem on the change of kinetic energy:

$mgx = \mu g \frac{M}{l}\frac{x^2}{2} + \frac{(M+m)V_{max}}{2}$, we find maximum velocity $\boldsymbol{V_{max}} \approx \mathbf{83}$ **cm/s**

Task 4

4.1 $\frac{\boldsymbol{Q}}{\boldsymbol{\Delta U}} = \mathbf{1}$

4.2 $\frac{\boldsymbol{V_3}}{\boldsymbol{V_1}} = \sqrt{\frac{\boldsymbol{T_3}}{\boldsymbol{T_1}}} = \mathbf{2}$

4.3 $\eta = 1 - \frac{Q_c'}{Q_h}$; $Q_c' = 2\nu R(T_3 - T_1)$; $Q_h = \frac{3}{2}\nu R(T_2 - T_1) + \frac{5}{2}\nu R(T_3 - T_2)$; $\boldsymbol{\eta} \approx \mathbf{7,7}$ **%**

Task 5

5.1 $E = k\frac{q}{r^2}$; $E_{r=2R} = k\frac{q}{4R^2}$; $E_{r=4R} = k\frac{q}{8R^2}$; $\frac{\boldsymbol{E_{r=2R}}}{\boldsymbol{E_{r=4R}}} = \mathbf{2}$

5.2 $\varphi = k\frac{q}{r} + \text{const}; \varphi_{sph} = \frac{2}{3}k\frac{q}{R}; \varphi_b = \frac{4}{3}k\frac{q}{R} \Rightarrow \frac{\boldsymbol{\varphi_b - \varphi_{sph}}}{\boldsymbol{\varphi_b}} = \mathbf{0,5}$

5.3 $\varphi'_b = \frac{2}{3}k\frac{q}{R}\left(\frac{\varepsilon+1}{\varepsilon}\right); \frac{\boldsymbol{\varphi_b}}{\boldsymbol{\varphi'_b}} = \frac{\mathbf{2}\boldsymbol{\varepsilon}}{\boldsymbol{\varepsilon}+\mathbf{1}} = \mathbf{1,2}$